2020 Maths Olympiads Division J Resource Kit C

This is the third of four kits in this special 2020 intra-school implementation of the APSMO Maths Olympiads.

At APSMO, we strongly believe that the ultimate goal of school mathematics, is to develop in our students the ability to solve problems. However, the current educational landscape presents a number of challenges for the implementation of problem solving teaching methods that we know students have used with considerable success. These methods rely largely on students having a go, explaining the strategies they used, and then learning from the strategies that were used by their peers.

In order to provide opportunities for such learning when teaching is being delivered remotely, we have selected a few problems from competitions from previous years. For each of these problems, a number of different solution methods are then suggested, so that students can still be exposed to multiple ways of approaching the problem. This leads to a recognition that solving the problem successfully can be achieved by applying logical and mathematical reasoning in a number of different ways.

Examples of how this kit may be used include:

- Introducing new or different solution methods;
- Providing diagrams that support a teacher's or student's explanations;
- Offering problem-solving homework (within this kit, there is a single page that includes all of the questions);
- Supporting students' own study as a stand-alone resource.

Further questions and solution methods can also be found in the APSMO resource books, available from www.apsmo.edu.au.

We hope that you will find this resource kit useful. Another kit will become available in the lead-up to 2020 Olympiad 5.

2020 Maths Olympiads Division J
Resource Kit C

C.1) Aaron, Becky and Chris are each wearing a shirt. One of them is wearing a pink shirt, one is wearing a yellow shirt, and one is wearing a white shirt (though not necessarily in that order).

Aaron is not wearing a yellow shirt.

Becky says to Chris, "I like your white shirt."

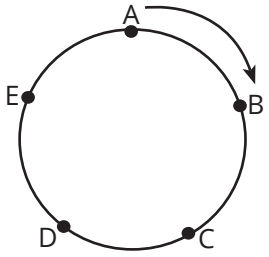
What colour is Becky wearing?

C.2) An ant travels around a circle in the direction shown.

It touches each of the labelled points in order.

The first three points that the ant touches are A, B, and C in that order.

What will be the 143rd point that the ant touches?



C.3) What number between 104 and 140 is divisible by both 6 and 15?

C.4) There is a total of 100 people on a plane.

There are 12 more women than men.

How many women are on the plane?

C.5) Suppose A and B represent any two numbers and A ✪ B = (A x A) – (B x B).

What is the sum of (8 ✪ 5) + (5 ✪ 2)?

Example Problem C.1

Aaron, Becky and Chris are each wearing a shirt. One of them is wearing a pink shirt, one is wearing a yellow shirt, and one is wearing a white shirt (though not necessarily in that order).

Aaron is not wearing the yellow shirt.

Becky says to Chris, "I like your white shirt."

What colour is Becky wearing?

Strategy : Eliminate all but one possibility

Let's draw a table with the possible combinations.

	Pink	Yellow	White
Aaron			
Becky			
Chris			

Aaron is not wearing the yellow shirt.

Let's mark this on the table with a cross, to show that it is not possible.

	Pink	Yellow	White
Aaron		×	
Becky			
Chris			

Becky says to Chris, "I like your white shirt".

So Chris must be wearing the white shirt.

Let's mark this with a tick, to show that it must be true.

	Pink	Yellow	White
Aaron		×	
Becky			
Chris			✓

If Chris is wearing the white shirt, he's not wearing pink or yellow. Aaron and Becky are not wearing white.

	Pink	Yellow	White
Aaron		×	×
Becky			×
Chris	×	×	✓

Aaron isn't wearing yellow.
Chris isn't wearing yellow.
So Becky must be wearing a yellow shirt and not pink.

	Pink	Yellow	White
Aaron		×	×
Becky	×	✓	×
Chris	×	×	✓

So Becky is wearing a **yellow** shirt.

Note: *You can introduce the logic problem table format by spelling out all of the possibilities, like this:*

Aaron wears	Pink?	Yellow?	White?
Becky wears	Pink?	Yellow?	White?
Chris wears	Pink?	Yellow?	White?

Students can then "invent" the more efficient table format for themselves

Answer: Yellow

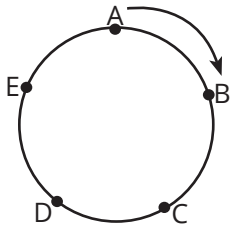
Example Problem C.2

An ant travels around a circle in the direction shown.

It touches each of the labelled points in order.

The first three points that the ant touches are A, B, and C in that order.

What will be the 143rd point that the ant touches?



Strategy 1: Count by complete circuits of the circle

Each circuit is 5 points.

It follows that to reach the 143rd point, the ant has made 143 divided by 5 circuits.

The ant travels 28 circuits with a remainder of 3 points.

The third point in the circuit is **C.**

Strategy 2: Solve a simpler related problem

We are told that the ant started walking from A to B to C to...

ABCDE ABCDE ABCDE ...

So, every 5th point the ant touches is an E, so the 140th point is an E.

The 141st point is A, the 142nd point is B, the 143rd point is C.

The 143rd point that the ant touches is a **C.**

A	B	C	D	E
1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
...				20
...				25
...				30
...				35
...				40
...				45
...				50
...	...	...	...	...
...	...	...	...	...
...				115
...				120
...				125
...				130
...				135
136	137	138	139	140
141	142	143		

Answer: C

Example Problem C.3

What number between 104 and 140 is divisible by both 6 and 15?

Strategy 1: Use the Lowest Common Denominator (LCM)

Which numbers are divisible by 6 and by 15?

The lowest common multiple of 6 and 15 is 30.

So, the number we are looking for must be a multiple of 30.

The only multiple of 30 between 104 and 140 is 120.

120 is divisible by 6 and by 15.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
														45
														60
														75
														90
														105
														120
														135
														150

Strategy 2: Make a list and compare multiples of 6 and 15.

Our number is between 104 and 140.

If we look at multiples of 15, we have: 15, 30, 45, 60, 75, 90, 105, 120, 135, 150, 165, ...

Now multiples of 6. We have: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96, 102, 108, 114, 120, 126, 132, 138, 144, 150, ...

Now compare the two lists:

~~15, 30, 45, 60, 75, 90,~~ 105, **120**, 135, ~~150, 165~~,...
~~6, 12, 18, 24, 30, 36, 42, 54, 60, 66, 72, 78, 84, 90, 96, 102,~~ 108, 114, **120**, 126, 132, 138, ~~144, 150~~, ...

The only number in both lists is **120**.

So, **120 is the only solution** that is a multiple of both 6 and 15**.**

Strategy 3: Use the divisibility rules

A number divisible by 6 must be even (divisible by 2) and divisible by 3.

Let's look at the numbers that are even.

Possible numbers are 106, 108, 110, 112, 114, 116, 118, 120, 122, 124, 126, 128, 130, 132, 134, 136, and 138.

Now, the divisibility test for 3 is to add the digits of the number and if the solution is divisible by 3, then the number is divisible by 3.

Numbers that are divisible by 2 and 3 are 108, 114, 120, 126, and 132.

Let's look at the multiples of 15 – they end in 5 or 0 – there are no numbers ending with 5 and only one ending with 0.

So, **120 is the only solution.**

Alternate method:

Another way to find the multiples of 15 is to add by 15s.

This would give the numbers 105, 120 and 135.

Only 120 is also a multiple of 6.

So, **120 is the only solution**.

Answer: 120

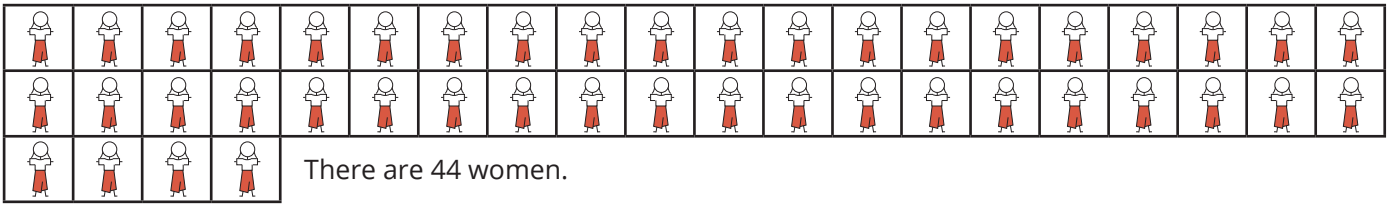
Example Problem C.4

There are 100 people on a plane. There are 12 more women than men. How many women are on the plane?

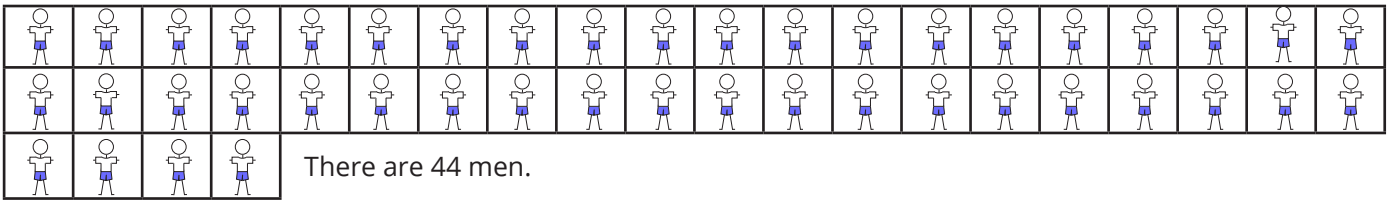
Strategy 1 : Pair each man with a woman using a diagram.

There were 100 people on the plane. There are 12 more women than men. We will ask 12 women to stand to one side so that the main group will have just as many women as men.
100 – 12 = 88. These 88 men and women can be paired.

Pair the men and women so there are 2 groups of 44 people.

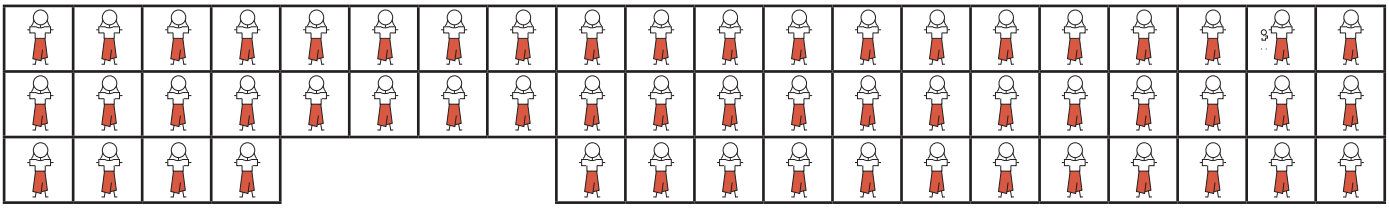


There are 44 women.



There are 44 men.

We have to add the extra 12 women who were not paired to the women who were paired.



There were (44 + 12) women on the plane. There were **56 women** on the plane.

Strategy 2 : Make a table

We start with 100 people, so suppose we have 50 men and 50 women.
There must be 12 fewer men than women. Change one man at a time.

	No. of women	No. of men	Difference between No. women and No. of men
	50	50	0
Move one man	51	49	2
Move another	52	48	4
Move another	53	47	6
Move another	54	46	8
Move another	55	45	10
Move another	56	44	12

We moved 6 men so there were 56 women and 44 men. There **were 56 women on the plane.**

Strategy 3 : Use algebra.

Let **m** be the number of men. Then **(m + 12)** is the number of women.

Altogether, there were 100 men and women	m + (m + 12) = 100
Combine all the terms	2m + 12 = 100
Subtract 12 from each side of the equation	2m = 88
Divide each side of the equation by 2	m = 44

Therefore, there are 44 men and (44 + 12) women. There were **56 women**.

Answer: 56

Example Problem C.5

Suppose A and B represent any two numbers and A ✪ B = (A x A) – (B x B).
What is the sum of (8 ✪ 5) + (5 ✪ 2)

Strategy 1: Evaluate each expression

(8 ✪ 5) = (8 x 8) – (5 x 5)
= 64 - 25
= 39

(5 ✪ 2) = (5 x 5) - (2 x 2)
= 25 - 4
= 21

Therefore, (8 ✪ 5) - (5 ✪ 2) = 39 +21
(8 ✪ 5) - (5 ✪ 2) = **60**

Strategy 2: Eliminate unnecessary terms

(8 ✪ 5) - (5 ✪ 2) = (8 x 8) - (5 x 5) + (5 x 5) - (2 x 2)

Now, we can see that (5 x 5) is both added and subtracted. – (5 x 5) + (5 x 5)

(8 x 8) – (5 x 5) + (5 x 5) – (2 x 2) = (8 x 8) – (~~5 x 5~~) + (~~5 x 5~~) – (2 x 2)
= (8 x 8) - (2 x 2)
= 64 - 4
= 60

(8 ✪ 5) - (5 ✪ 2) = **60**

Answer: 60

Answers to Example Problems

C.1	**Yellow**
C.2	**C**
C.3	**120**
C.4	**56**
C.5	**60**