2020 Maths Olympiads Division J
Resource Kit D

MATHS OLYMPIAD

This is the last of four kits in this special 2020 intra-school implementation of the APSMO Maths Olympiads.

At APSMO, we strongly believe that the ultimate goal of school mathematics, is to develop in our students the ability to solve problems. However, the current educational landscape presents a number of challenges for the implementation of problem solving teaching methods that we know students have used with considerable success. These methods rely largely on students having a go, explaining the strategies they used, and then learning from the strategies that were used by their peers.

In order to provide opportunities for such learning when teaching is being delivered remotely, we have selected a few problems from competitions from previous years. For each of these problems, a number of different solution methods are then suggested, so that students can still be exposed to multiple ways of approaching the problem. This leads to a recognition that solving the problem successfully can be achieved by applying logical and mathematical reasoning in a number of different ways.

Examples of how this kit may be used include:

- Introducing new or different solution methods;
- Providing diagrams that support a teacher's or student's explanations;
- Offering problem-solving homework (within this kit, there is a single page that includes all of the questions);
- Supporting students' own study as a stand-alone resource.

Further questions and solution methods can also be found in the APSMO resource books, available from www.apsmo.edu.au.

We hope that you will find this final resource kit useful.

2020 Maths Olympiads Division J Resource Kit D

D.1) Lauren went shopping with all her birthday money.

She spent half of it on a pair of jeans, a third of what was left on a T-shirt, and a sixth of what was left after that on a slice of pizza and a can of soft drink.

She returned home with $25.

How much money did she receive for her birthday?

D.2) I am thinking of a number.

If you subtract 3 from my number and then multiply by 4, the result is 28.

What is the number I am thinking of?

D.3) Each number from 1 to 16 is written in a square.

A path is formed by placing consecutive numbers in adjacent boxes horizontally or vertically, but not diagonally.

Four numbers are shown.

Find the sum of the numbers in the starred (★) boxes.

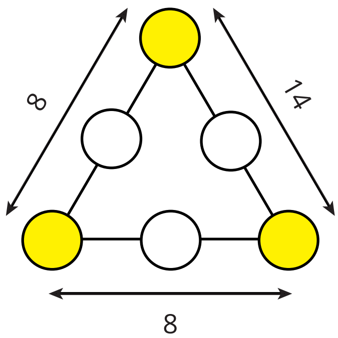
3			
	1		
5	★		
		16	★

D.4) Each of the 6 circles contains a different counting number.

The sum of the 6 numbers is 21.

The sum of the 3 numbers along each side of the triangle is shown in the diagram.

What is the sum of the numbers in the shaded circles?



D.5) Four volunteers can pack 12 boxes every 30 minutes.

How many volunteers are needed to pack 72 boxes every hour?

Assume all volunteers work at the same rate.

Example Problem D.1

Lauren went shopping with all her birthday money.

She spent half of it on a pair of jeans, a third of what was left on a T-shirt, and a sixth of what was left after that on a slice of pizza and a can of soft drink.

She returned home with $25.

How much money did she receive for her birthday?

Strategy : Work backwards with a diagram.

We know that Lauren received birthday money

She spent $\frac{1}{2}$ of that money on jeans

She has $\frac{1}{2}$ left

She spent $\frac{1}{3}$ of what was left on a T-shirt

$\frac{1}{3}$ of $\frac{1}{2}$ left = $\frac{1}{6}$ of original money

She has spent $\frac{1}{2} + \frac{1}{6} = \frac{6}{12} + \frac{2}{12} = \frac{8}{12} = \frac{2}{3}$

She has $\frac{1}{3}$ left

She spent $\frac{1}{6}$ of what was left on pizza & drink

$\frac{1}{6}$ of $\frac{1}{3}$ left = $\frac{1}{18}$ of original money

She has spent $\frac{1}{2} + \frac{1}{6} + \frac{1}{18} = \frac{9}{18} + \frac{3}{18} + \frac{1}{18} = \frac{13}{18}$

She has $\frac{5}{18}$ left

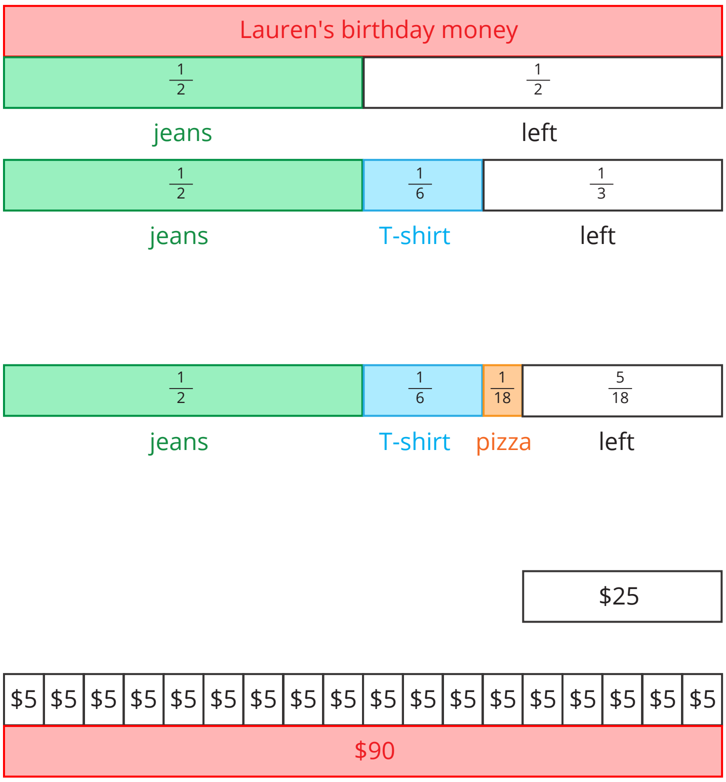
Lauren has $25 left

If $\frac{5}{18}$ of the money = $25

Then $\frac{1}{18}$ of the money = $5

$5 x 18 = $90

Therefore, Lauren received **$90** in birthday money.



Check:

If Lauren received $90, then

$\frac{1}{2}$ of the money = $45 spent on jeans

So, Lauren has $90 - $45 = $45 left

$\frac{1}{3}$ of what was left = $\frac{1}{3}$ of $45 = $15 on the T-shirt

So, Lauren has $45 - $15 = $30 left

$\frac{1}{6}$ of what was left = $\frac{1}{6}$ of $30 = $5 on pizza and drinks

She has $30 - $5 = $25 left.

So, Lauren had **$90** in birthday money.

Answer: $90

Example Problem D.2

I am thinking of a number.

If you subtract 3 from my number and then multiply by 4, the result is 28.

What is the number I am thinking of?

Strategy 1: Work backwards, using opposite operations.

Question: My number −3 → ? → × 4 → 28

Solution: 10 ← +3 ← 7 ← ÷ 4 ← 28 (↑ Start here)

The result is (28 ÷ 4) + 3 = 10. My number is **10**.

> ***Let's check:***
>
> 10 - 3 = 7 then 7 x 4 = 28.

Strategy 2: Use algebra.

Let *N* represent my number,

The equation to solve is $4\,(N - 3) = 28$

Divide both sides of the equation by 4 $(N - 3) = 7$

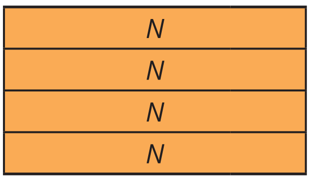
Add 3 to each side of the equation $N = 10$

My number is **10**.

Strategy 3: Use a diagram

Start with the number *N*

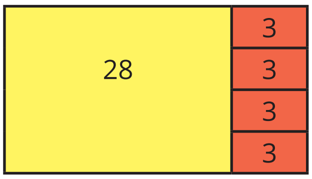
Since multiplication is like repeated addition, we can represent 4 x *N* like this:



If we subtract 3 from *N*,

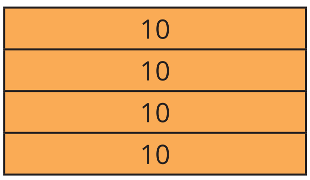
then 4 x (*N* - 3) would look like this:

We know that the result of 4 x (*N*- 3) = 28



Therefore,
4 x *N* = 28 + (4 x 3) = 40,
and so *N* = 40 ÷ 4 = 10

My number is **10**.



Answer: 10

Example Problem D.3

Each number from 1 to 16 is written in a square.

A path is formed by placing consecutive numbers in adjacent boxes horizontally or vertically, but not diagonally.

Four numbers are shown.

Find the sum of the numbers in the starred (✪) boxes.

3			
	1		
5	✪		✪
		16	

Strategy 1: Start where there is only one possible choice.

The square between 3 and 5 must contain 4.

There is then only one possible location for 2 that lies adjacent to both 1 and 3.

3	2		
4	1		
5	✪		✪
		16	

There are 2 possible locations for 6.

If 6 is placed to the right of 5, the square below the 6 will be 7 and 8 will be in the left-hand corner.

However, the lower left corner will then be a dead end (how can we place 9?).

So, ✪ cannot be 6.

3	2		
4	1		
5	6		✪
8	7	16	

So, 6 is placed in the lower left corner.

It follows that 7 is between the 6 and 16.

8 replaces the left hand ✪ and 9 will be between the two ✪.

We then put 10 then 11 in the column above the 9.

The 12, 13, 14 and 15 complete the right-hand column.

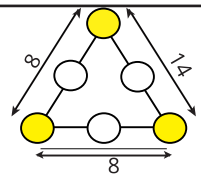
3	2	11	12
4	1	10	13
5	8	9	14
6	7	16	15

Therefore, the sum of the numbers in the starred boxes is 8 + 14 = **22**.

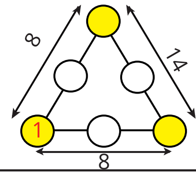
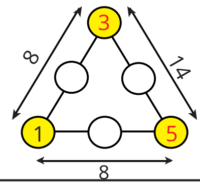
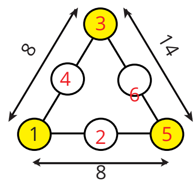
Answer: 22

Example Problem D.4

Each of the 6 circles contains a different counting number.
The sum of the 6 numbers is 21.
The sum of the 3 numbers along each side of the triangle is shown in the diagram.
What is the sum of the numbers in the shaded circles?



Strategy 1: Determine the numbers and then their placement.

The smallest possible sum for 6 different counting numbers:	1 + 2 + 3 + 4 + 5 + 6 = 21
The sum of the 6 numbers is given as 21, so the numbers must be:	1, 2, 3, 4, 5, and 6
Two given sums are each 8 and the only sets of these numbers with that sum are:	{1, 3, 4} and {1, 5, 2}
Both sets of numbers have 1, so put 1 in the intersection of the two sides that equal 8.	
{1, 3, 4} and {1, 5, 2} These numbers are in the five circles along the left and bottom sides of the triangle. We have to decide which numbers go in the coloured circles, so the third side adds to 14.	
If we put the 3 and 5 in the coloured circles then we can put a 6 in the middle circle on that side. The total is 14. We can put the other numbers for the 8 on the other two sides. **The three shaded circles contain the numbers 1, 3 and 5, and their sum is 9**.	

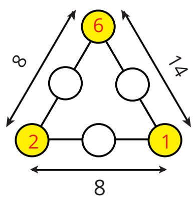
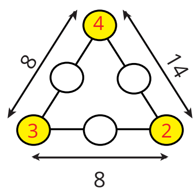
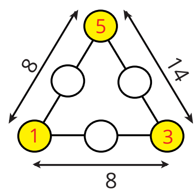
Strategy 2: Find the sum of the numbers along each side.

The sum of the first 6 counting numbers is 21.

If we add together the sums along each side, we will end up counting each of the shaded circles twice.

Since 8 + 8 + 14 = 30, and the sum of the 6 numbers is 21, the sum of the numbers in the three shaded circles must be 30 - 21 = **9**.

Think about the combinations of numbers that equal 9: {6, 2, 1}, {5, 3, 1}, {4, 3, 2}.

		
{6, 2, 1} If we put {6, 2, 2} in the shaded circles, then we have 3, 4, and 5 to go in the open circles. We can try all combinations but none make the totals given in the question.	{4, 3, 2} If we put {3, 4, 2} in the shaded circles, then we have 1, 4, and 6 to go in the open circles. We can try all combinations but none make the totals given in the question.	{5, 3, 1} If we put {5, 3, 1} in the shaded circles, then we have 2, 4, and 6 to go in the open circles. We can try all combinations and find that: Between 5 and 3, we put 6 to get 14. Between 1 and 3, we put 4 to get 8. Between 1 and 5, we put 2 to get 8.

Therefore, all six numbers are used and the shaded numbers are {5, 3, 1} and the sum is **9.**

Answer: 9

Example Problem D.5

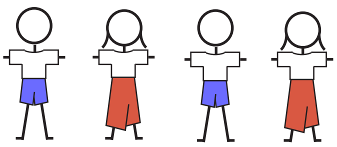
Four volunteers can pack 12 boxes every 30 minutes.

How many volunteers are needed to pack 72 boxes every hour?

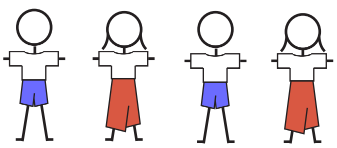
Assume all volunteers work at the same rate.

Strategy 1: Find how many boxes 4 volunteers can pack in 1 hour.

If 4 volunteers pack 12 boxes every 30 minutes, they can pack 2 x 12 = 24 boxes in one hour.



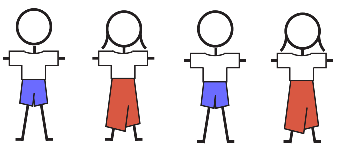
Box 1	Box 2	Box 3	Box 4
Box 5	Box 6	Box 7	Box 8
Box 9	Box 10	Box 11	Box 12
30 minutes			



Box 1	Box 2	Box 3	Box 4	Box 13	Box 14	Box 15	Box 16
Box 5	Box 6	Box 7	Box 8	Box 17	Box 18	Box 19	Box 20
Box 9	Box 10	Box 11	Box 12	Box 21	Box 22	Box 23	Box 24
30 minutes				30 minutes			
1 hour							

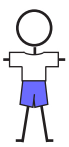
Since 4 volunteers can pack 24 boxes per hour, we need 3 times as many volunteers to pack 72 boxes in an hour. Therefore, we need 3 x 4 = **12** volunteers to pack 72 boxes in an hour.

Strategy 2: Find how many boxes 1 volunteer can pack in an hour.



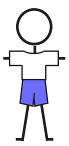
Box 1	Box 2	Box 3	Box 4
Box 5	Box 6	Box 7	Box 8
Box 9	Box 10	Box 11	Box 12
30 minutes			

4 volunteers can pack 12 boxes every 30 minutes.



Box 1	Box 2	Box 3	Box 4
Box 5	Box 6	Box 7	Box 8
Box 9	Box 10	Box 11	Box 12
30 minutes			

1 volunteer can pack $\frac{1}{4}$ as many boxes in 30 minutes, or 3 boxes.



Box 1				Box 4			
Box 2				Box 5			
Box 3				Box 6			
30 minutes				30 minutes			
1 hour							

One volunteer can pack 6 boxes in one hour.

Therefore, as 72 ÷ 6 = 12, we need **12** volunteers to pack 72 boxes in an hour.

Answer: 12

Answers to Example Problems

D.1	**$90**
D.2	**10**
D.3	**22**
D.4	**9**
D.5	**12**